Course Notes
for
MA4601
Science Mathematics 1

J. Kinsella

October 11, 2000

1 PerpDist

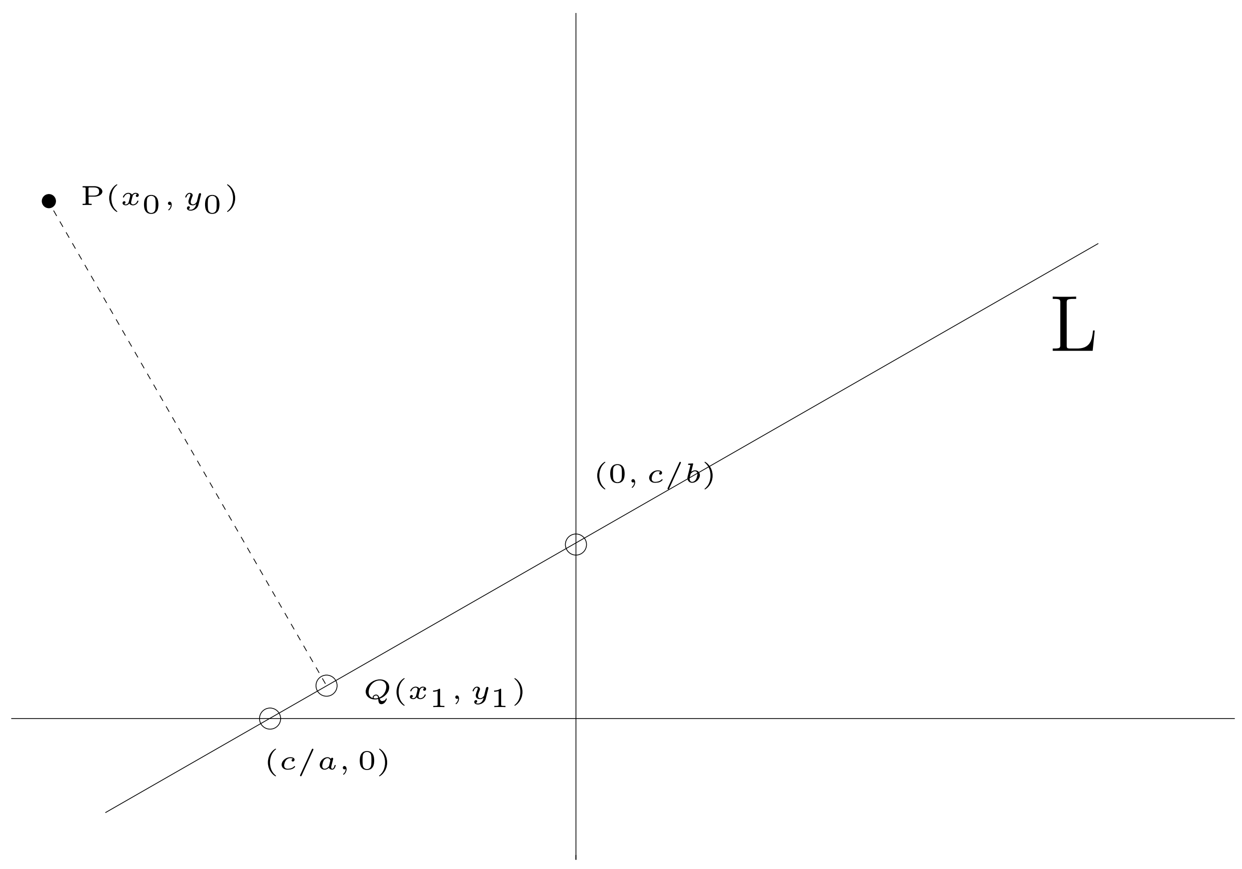

Figure 1.1: Perpendicular Distance from a point to a line

Want $\perp$ distance from point $P(x_0, y_0)$ to line $L : ax + by = c$. Let $Q(x_1, y_1)$ be the intersection of the perpendicular from P with the line L, as in the Figure. Clearly the vector $\underline{PQ}$ is $\perp L$ and therefore to the vector $\langle -c/a, c/b \rangle$ which points from the x–intercept to the y–intercept.

So

$$\langle x_0 - x_1, y_0 - y_1 \rangle \perp \langle -c/a, c/b \rangle,$$

or (multiplying the latter vector by ab/c), we have:

$$\langle x_0 - x_1, y_0 - y_1 \rangle \perp \langle -b, a \rangle.$$

We also have of course that

$$ax_1 + by_1 = c.$$

The desired distance $d = |\underline{PQ}|$ is just:

$$d = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}.$$

So calling

$$\Delta x = (x_0 - x_1) \quad \text{and} \quad \Delta y = (y_0 - y_1)$$

we have the two equations:

$$\begin{aligned} -b\Delta x + a\Delta y &= 0 \\ a\Delta x + b\Delta y &= ax_0 + by_0 - c = T \quad \text{(say)}. \end{aligned}$$

Solving we find:

$$\begin{aligned} \Delta x &= \frac{aT}{a^2 + b^2} \\ \Delta y &= \frac{bT}{a^2 + b^2}. \end{aligned}$$

Substituting into

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

gives the result:

$$d = \frac{|T|}{\sqrt{a^2 + b^2}} = \frac{|ax_0 + by_0 - c|}{\sqrt{a^2 + b^2}}.$$

Whew....