UNIVERSITY *of* LIMERICK

OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4601

SEMESTER: Autumn 2002

MODULE TITLE: Science Mathematics 1

DURATION OF EXAMINATION: 2 1/2 hours

LECTURER: Dr. J. Kinsella

PERCENTAGE OF TOTAL MARKS: 80%

EXTERNAL EXAMINER: Prof. J.D. Gibbon

INSTRUCTIONS TO CANDIDATES: **Answer four questions correctly for full marks — 80%.**

1 (a) Given the vectors $\mathbf{a} = \langle -1, 1, 1\rangle$ and $\mathbf{b} = \langle 3, 2, 1\rangle$, find

(i) $|\mathbf{a}|$, 1

(ii) $|\mathbf{b}|$, 1

(iii) $\mathbf{a} \cdot \mathbf{b}$, 1

(iv) the angle between $\mathbf{a}$ and $\mathbf{b}$. (No calculator needed.). 1

(b) (i) Show that if θ is the angle between two vectors $\mathbf{a}$ & $\mathbf{b}$ in V^3 (so that $0 \leq \theta \leq \pi$), then 10

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta.$$

(ii) Find the area of the triangle PQR where $P = (1, 2, 3)$, $Q = (3, 1, 2)$ and $R = 2, 3, 1)$. 8

(iii) Find a vector perpendicular to the plane in which the triangle PQR lies. 3

2 (a) Using either Gauss Elimination or Gauss-Jordan elimination, solve the linear system: 16

$$\begin{aligned} x+8y \quad &= 17 \\ 4y+z &= 11 \\ x+8y+z &= 20 \end{aligned}$$

(b) Find the determinant of the matrix 6

$$A = \begin{bmatrix} 1 & 8 & 0 \\ 0 & 4 & 1 \\ 1 & 8 & 1 \end{bmatrix}$$

(c) What is the significance of the value you found for the determinant of A in 2(b)? 3

3 It can be shown that, given two columns of numbers $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$, the slope m and the y–intercept c of the least squares best fit straight line $y = mx + c$ satisfy the matrix equation

$$\begin{bmatrix} n & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \begin{bmatrix} c \\ m \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}.$$

(a) A researcher monitors the size P of a population of animals as the foodstock available varies. Suppose that F is the amount of food available above or below some reference value — so F can be negative or positive. Find the least squares straight line fit to the data in the form $P = mF + c$. 20

F	**P**
-3	3
-2	5
-1	5
0	11
1	12
2	14
3	16

(b) Use the least squares fit found in part 3(a) above to predict the size of the population for $F = 4.5$. 5

4 (a) Calculate the following limits from first principles:

(i) $\lim_{x\to a} \dfrac{x^2 - a^2}{x - a}$, 2

(ii) $\lim_{x\to a} \dfrac{x^3 - a^3}{x - a}$. 3

(b) Calculate the derivative from first principles of the following functions at the indicated values of x:

(i) $f(x) = \frac{1}{x}$ at $x = a$. 3

(ii) $f(x) = \sqrt{x}$ at $x = a$, 4

(c) Prove the Quotient Rule: If f, g are differentiable at a and $g(a) \neq 0$ then 10

$$\left(\frac{f}{g}\right)'(a) = \frac{g(a)f'(a) - f(a)g'(a)}{g(a)^2}$$

(d) Differentiate the function f, where 3

$$f(x) = \frac{x\sin(x)}{1 + x^2\cos(x)}$$

5 (a) Find (if they exist) the: 15

- roots
- critical points (maximum/minimum points & points of inflection)
- vertical asymptotes
- horizontal/diagonal asymptotes

of the function $1 + \frac{(x-3)(x-2)}{x-1}$.

(b) Sketch the graph of f, using the information assembled in part (a). 10

6 A cone-shaped paper cup is to be made from a circular piece of paper with radius R by cutting a sector subtending an angle $2\pi - \theta$ from the disc. (See Fig. 1.)

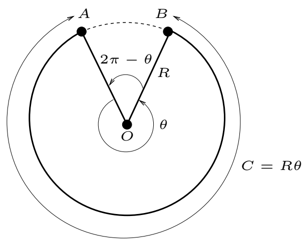

Figure 1: Disk with sector cut out.

The two sides OA and OB are now joined to make a cone. The circumference of the base of the cone is C and its slant height is R. (See Fig. 2.)

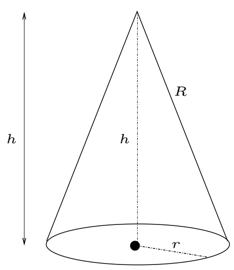

Figure 2: Cone-shaped cup (upside-down).

Now, using the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone — where r is the radius of the base of the cone — find the value of θ that maximises the volume of the cone. 25

Hint: the circumference of the base of the cone is just $C = R\theta$ so the radius of the base is $r = \frac{\theta}{2\pi}R$.