UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4601

SEMESTER: Autumn 2001

MODULE TITLE: Science Mathematics 1

DURATION OF EXAMINATION: 2 1/2 hours

LECTURER: Dr. J. Kinsella

PERCENTAGE OF TOTAL MARKS: 80%

EXTERNAL EXAMINER: Prof. J.D. Gibbon

INSTRUCTIONS TO CANDIDATES: **Answer four questions correctly for full marks — 80%.**

1 (a) Given the vectors $\mathbf{a} = \langle 0, 1, 7 \rangle$ and $\mathbf{b} = \langle 1, 0, 4 \rangle$, find

(i) $|\mathbf{a}|$, 1

(ii) $|\mathbf{b}|$, 1

(iii) $\mathbf{a} \cdot \mathbf{b}$, 1

(iv) the angle between $\mathbf{a}$ and $\mathbf{b}$. (No calculator needed.). 2

(b) (i) Refer to Figure 1 below. Using the definitions of the vectors $\mathbf{a}$ and $\mathbf{b}$ given above, find a formula for the scalar projection of $\mathbf{a}$ along $\mathbf{b}$. 3

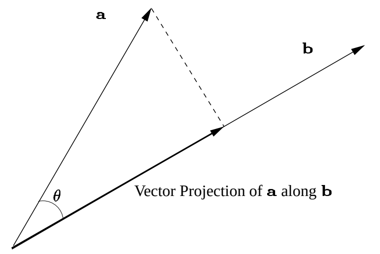

Figure 1: Scalar/Vector Projection of $\mathbf{a}$ along $\mathbf{b}$

(ii) Find a formula for the vector projection of $\mathbf{a}$ along $\mathbf{b}$. 2

(c) (i) Use a scalar projection to show that the perpendicular distance from a point $P(x_1, y_1)$ to the line $ax + by + c = 0$ is: 12

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Illustrate your proof with a simple sketch based on Figure 2

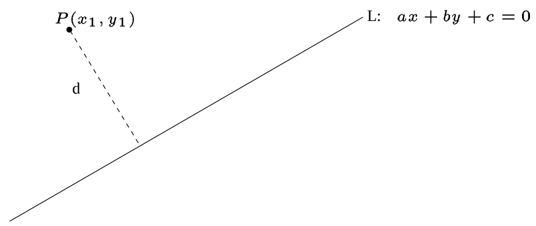

Figure 2: Perpendicular Distance from P to L

(ii) Use the formula proved in part 1(c)(i) above to find the perpendicular distance from the point $(1, 2)$ to the straight line $3x + 2y = 1$. 3

2 (a) Using either Gauss Elimination or Gauss-Jordan elimination, solve the linear system: 15

$$\begin{aligned} x - 3y + \ z &= 5 \\ 4y + 8z &= 4 \\ -2x + 6y + 7z &= -1 \end{aligned}$$

(b) Find the determinant of the matrix 12

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 0 & 4 & 8 \\ -2 & 6 & 7 \end{bmatrix}$$

(c) What is the significance of the value you found for the determinant in 2(b)? 3

3 It can be shown that, given two columns of numbers $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$, the slope m and the y–intercept c of the least squares best fit straight line $y = mx + c$ satisfy the matrix equation

$$\begin{bmatrix} n & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \begin{bmatrix} c \\ m \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}.$$

(a) A chemical used to control bacteria in a water plant is tested for effectiveness. In the following table, x represents the number of litres of the chemical in the pool (above or below the "standard amount") and y represents the corresponding bacteria count in thousands.

x	-3	-2	-1	0	1	2	3
y	5	4.5	4	3.5	3	2.5	2

Find the least squares straight line fit to the data in the form $y = mx + c$. 20

(b) Use the least squares fit found in part 3(a) above to predict the bacteria count (y) when $x = 4$. 5

4 (a) Calculate the following limits from first principles — state briefly any properties of limits that you use:

(i) $\lim_{x \to a} \frac{x^2 - a^2}{x + a}$, 2

(ii) $\lim_{x \to a} \frac{x^2 - a^2}{x - a}$. 3

(b) Calculate the derivative from first principles of the following functions at the indicated values of x:

(i) $f(x) = x^2$ at $x = a$, 5

(ii) $f(x) = \frac{1}{x}$ at $x = a$. 5

(c) Prove the Chain Rule: If a function g is differentiable at a and a function f is differentiable at $g(a)$, then the function $f \circ g$ is differentiable at a and: 7

$$(f \circ g)'(a) = f'(g(a)).g'(a)$$

(d) Use the Chain Rule to differentiate the function f, where 3

$$f(x) = \sin\left(\frac{1}{\cos(x^2)}\right)$$

5 (a) Find (if they exist) the: 15

- critical points (maximum/minimum points & points of inflection)
- roots
- vertical asymptotes
- horizontal/diagonal asymptotes

of the function $f(x) = \frac{(x-1)(x-2)}{2x-1}$.

(b) Sketch the graph of f, using the information assembled in part (a). 10

6 A rain gutter is to be constructed from a metal sheet of width 30 cm. by bending up 10 cm. of the sheet on each side (see Figure 3 below) through an angle θ. What is the value of θ that maximises the amount of water contained in the gutter. 25

Hint: treat the problem as two-dimensional. The problem reduces to finding the angle θ that maximises the area of the plane figure A-B-C-D given that $|AB| = |BC| = |CD| = 10$cm..

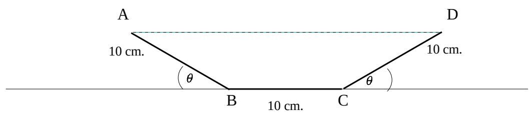

Figure 3: Fill the gutter to the dotted line.