UNIVERSITY *of* LIMERICK
OLLSCOIL LUIMNIGH

College of Informatics and Electronics

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4601

SEMESTER: Autumn 2000

MODULE TITLE: Science Mathematics 1

DURATION OF EXAMINATION: 2 1/2 hours

LECTURER: Dr. J. Kinsella

PERCENTAGE OF TOTAL MARKS: 90%

EXTERNAL EXAMINER: Prof. J.D. Gibbon

INSTRUCTIONS TO CANDIDATES: **Answer four questions correctly for full marks — 90%.**

1 (a) Given the vectors $\mathbf{a} = \langle 2, 1, 3\rangle$ and $\mathbf{b} = \langle 1, 7, 2\rangle$, find

(i) the magnitude of $\mathbf{a} + \mathbf{b}$, 2

(ii) a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. 3

(b) (i) Show using the Triangle Law for addition of vectors that any point $Q(x, y, z)$ on a line L in three dimensions has position vector q which satisfies

$$q = q_0 + t * (q_1 - q_0),$$

where q_0 and q_1 are the position vectors of any two given points Q_0 and Q_1 on the line and t can take any real value. 5

(ii) Use this result to find the equation of the line containing the points $Q_0(2, 1, 3)$ and $Q_1(1, 7, 2)$ in parametric form (i.e. expressing x, y and z in terms of t). 5

(c) (i) Show that the perpendicular distance from a point P to the line containing any two given points Q_0 and Q_1 is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{b}|},$$

where $\mathbf{a} = \overrightarrow{Q_0P} = \mathbf{p} - \mathbf{q_0}$ and $\mathbf{b} = \overrightarrow{Q_0Q_1} = \mathbf{q_1} - \mathbf{q_0}$. 5

(ii) Use this result to find the perpendicular distance from the point $P(1, 4, 2)$ to the line containing the points $Q_0(-1, 3, 1)$ and $Q_1(-2, 1, 4)$ 5

2 (a) Using either Gauss Elimination or Gauss-Jordan elimination, solve the linear system: 7

$$\begin{aligned} x + 2y + \ 2z &= 1 \\ 4y + \ 5z &= 2 \\ 3x + 6y + \ 7z &= 1 \end{aligned}$$

(b) (i) Set up the linear system in (a) as a matrix equation of the form $AX = b$. 2

(ii) Find the inverse of A. 5

(iii) Use the result found in (ii) to solve the matrix equation above for X. 4

(c) (i) Find the determinant of A. 5

(ii) What conclusion may we draw from the fact that its value is not zero? 2

3 (a) Show that, given two columns of numbers $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$, 13

the slope m and the y–intercept c of the least squares best fit straight line $y = mx + c$ satisfy the matrix equation

$$\begin{bmatrix} n & \sum x \\ \sum x & \sum x^2 \end{bmatrix} \begin{bmatrix} c \\ m \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}.$$

(b) Using the matrix equation discussed in (a), find the least squares best fit straight line $y = mx + c$ for the following data: 12

x	y
-3	12
-2	11
-1	10
0	4
1	-1.5
2	-2
3	-6

4 (a) Calculate the following limits from first principles:

(i) $\displaystyle\lim_{x\to a} \frac{x^3 - a^3}{x + a}$, 2

(ii) $\displaystyle\lim_{x\to a} \frac{x^3 - a^3}{x - a}$. 3

(b) Calculate the derivative from first principles of the following functions at the indicated values of x:

(i) $f(x) = x^3$ at $x = a$, 5

(ii) $f(x) = \sqrt{x}$ at $x = a$. 5

(c) Prove the Quotient Rule, i.e. that if f and g are two functions which are differentiable at $x = a$ then, (provided $g(a) \neq 0$), 10

$$(f/g)'(a) = \frac{g(a)f'(a) - f(a)g'(a)}{g(a)^2}$$

5 (a) Find the: 10

- vertical asymptotes
- roots
- critical points (maximum/minium points & points of inflection)
- horizontal/diagonal asymptotes

of the function $f(x) = \dfrac{x^2+1}{x^2-4\,x+3}$.

(b) Sketch the graph of f using the information gathered in (a). 15

6 Consider the following problem (see the Figure). A man launches his boat from a point A on the bank of a river and wants to reach point C, 8 km. downstream on the opposite bank, as soon as possible. He could row directly across the river to P & run along the bank to C; he could row directly to C; or he could row to some intermediate point B and run along the bank to C. If he can row at 6 kph. and run at 8 kph., where should he land to reach C as soon as possible? (We assume that the current in the river is negligible.) 25

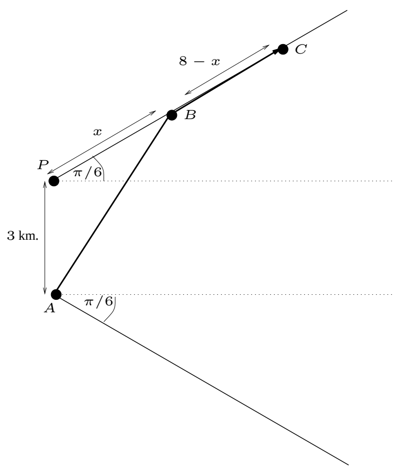

Figure 1: River Crossing

The complicating factor is that the river is not of constant width. As shown in the Figure, the banks of the river diverge at an angle of $\pi/6$ radians or $30°$. The width of the river at the narrowest point (A) is 3 km. (Hint: Consider the triangle APB and use the Cosine Rule for Triangles to find the distance AB.)